

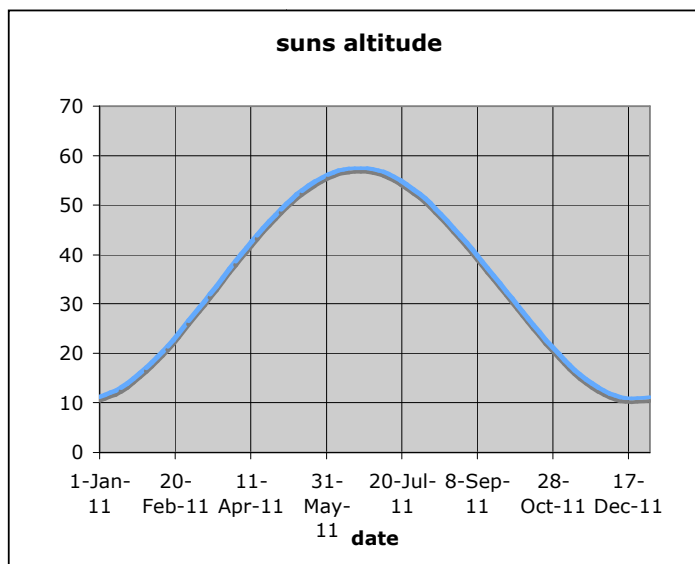
On the Pitch (teachers' notes)

Each Query should occupy a visit to the maths department; the students should do Query 1 without being privy to the content of subsequent queries.

Query 1:

At what angle would you recommend the pitch of the roof be set to get the best out of the roof over the year?

- (i) Draw a graph of *sun's altitude* against *time of year*.



Note 1: This could be a good opportunity to get the pupil au fait with the graphing potential of Excel.

Note 2: If you wish the x-axis to run from Jan 1 to Dec 31 (as above) then you need to know that, when setting the scale, the minimum value, corresponding to Jan 1, is 39082 and the maximum value corresponding to Dec 31 is 39446.

- (ii) Calculate the mean value of a° over the year.

$$\text{Average} = \text{total} \div \text{number of obs} = 839.6 \div 25 = 33.6 \text{ (to 1 d.p.)}$$

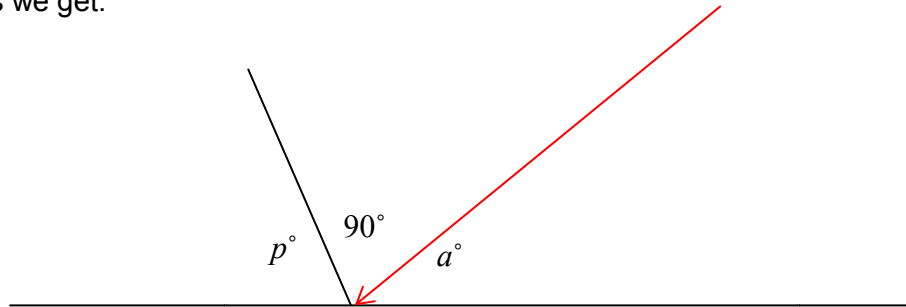
Note 1: The line representing the average can be drawn on the graph to add to argument. This is best done at the time of creating the 'altitude' graph if you are using Excel. This ensures the line is an integral part of the graph, and shouldn't be left behind in the cut-n-paste process.

Note 2: This is leading up to a first approximation for a suitable pitch and is only considering one variable/aspect of the problem. It may be worth drawing this to the student's attention, that in the modeling process, very simple models are considered first and then complexity is added a variable at a time.

- (iii) **Make a case for a particular value of p° as being the most efficient in the context.**

We want the sun's rays to be perpendicular to the plane of the roof for most efficient output.

Drawing this we get:



The geometry tells us that p° is the complement of a° when the rays are perpendicular to the plane of the roof.

It would seem reasonable that picking the mean value of the sun's altitude would give the best compromise of the variable viz 33.6° ; thus the optimum value for the pitch would be the complement of this viz 55.4° .

Note: the latitude of Glasgow G1 is about 55.8°

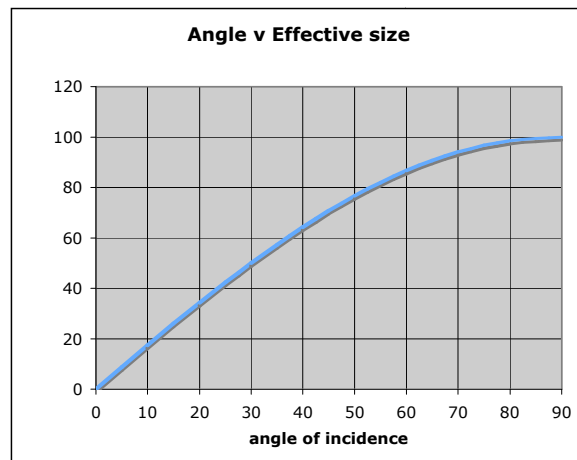
Query 2:

When a solar panel is in position AB and the rays of the sun strike it at x° , the effective size of the panel is actually A'B.

(i) Explore how the length of A'B varies as x varies. [Take AB = 100 units]

At level 4 a student should be able to make scale drawings for values of x , $0 \leq x \leq 90$ in jumps of 10° to get a reasonable table of values and hence a good graph of the relationship.

x	A'B
0	0
10	17.36481777
20	34.20201433
30	50
40	64.27876097
50	76.60444431
60	86.60254038
70	93.96926208
80	98.4807753
90	100



At the right time this could be used to introduce trigonometry.

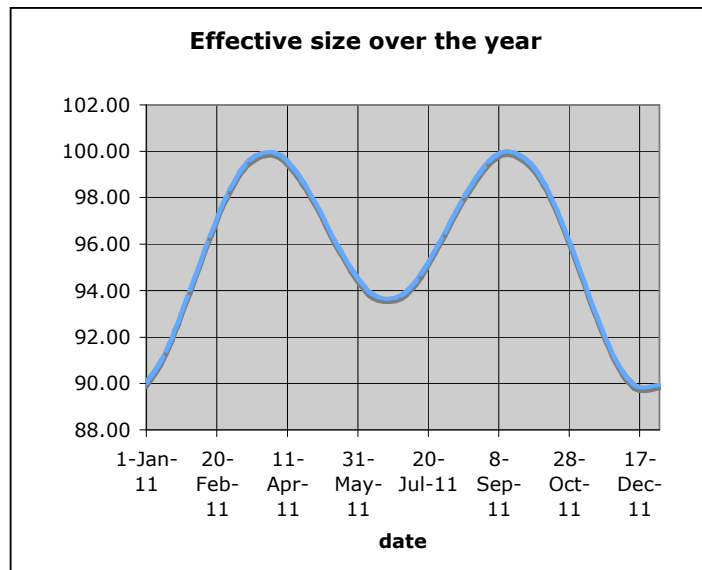
(ii) Explore how this relates to the context.

Simple geometry leads to the fact that $x = 180 - (p + a)$.

Using the table generated above, or interpolating using the graph, we can develop a table of *effective size* versus *time of year*.

One could link with *Geography* here by considering the equinoxes and the solstices.

The chart below has been calculated using an Excel spreadsheet and drawn from the data produced (see table below chart.)



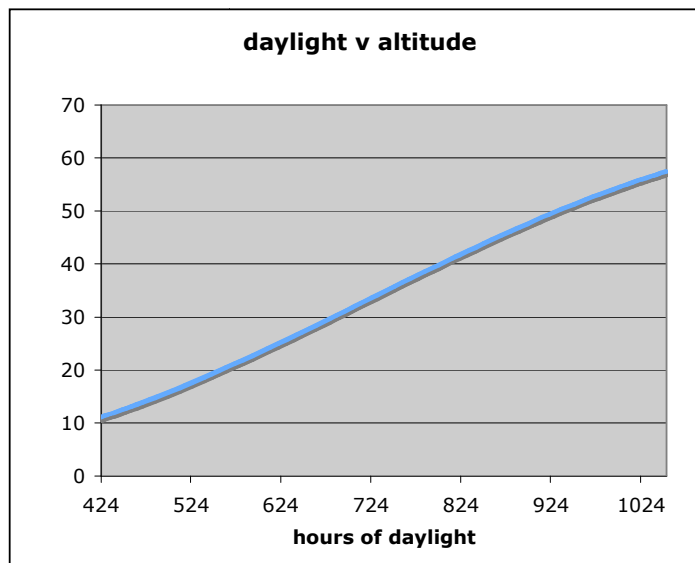
Query 3

The table below gives the amount of daylight in each day in the year in Glasgow.

- (i) Use it to explore the relationship between **amount of daylight** and **altitude**.

Date	effective size
1-Jan-11	87.63
15-Jan-11	89.18
1-Feb-11	92.12
15-Feb-11	94.83
1-Mar-11	97.28
15-Mar-11	99.03
1-Apr-11	99.97
15-Apr-11	99.77
1-May-11	98.71
15-May-11	97.44
1-Jun-11	96.03
15-Jun-11	95.42
1-Jul-11	95.48
15-Jul-11	96.27
1-Aug-11	97.74
15-Aug-11	98.98
1-Sep-11	99.91
15-Sep-11	99.88
1-Oct-11	98.77
15-Oct-11	96.90
1-Nov-11	93.85
15-Nov-11	91.14
1-Dec-11	88.62
15-Dec-11	87.46
31-Dec-11	87.55

Using a spreadsheet we can generate this chart which shows that there is a positive relationship between minutes of daylight and increasing altitude.



- (ii) **Why might you want the pitch to relate more to altitudes at times when the altitude takes bigger values?**
Might this lead you to reassessing the value of pitch you recommended in Query 1?

The higher the altitude the more hours of daylight there are.
 Rather than picking the mean value of the altitude as a basis for fixing the pitch, it would be more useful if the pitch were set to make most use of the extra daylight hours which occur at times of the sun's greatest altitude.

This would definitely affect the choice made in Q1, and query 4 will consider this.

Query 4

Suppose we use the product, *daylight times effective size* as a measure of the energy produced.

- (i) **Use a spreadsheet to help you explore the pattern of energy output over the year for different pitches.**
[These are better examined using the 'Chart' wizard.]
Comment on the pros and cons of each.

Note 1: The spreadsheet that can be used for this query is based on all the data that the student is already exposed to. However, in order to investigate relationships between pitch and energy the teacher will have to provide the student with one formula that is otherwise beyond the criteria of the level.

This formula is for column 4 which calculates the **effective size** given the pitch ... which is entered in A2. This formula requires elementary trigonometry and, because we are working with a spreadsheet, radians!

(References to cells are for the sheet given below)
Assuming the word Pitch is in A1 then the formula

$$=100*\text{SIN}(\text{RADIANS}(\$A\$2+D2))$$

should be put in cell E2 and filled down ... This will calculate **effective sizes** for the given pitch for the different altitudes.

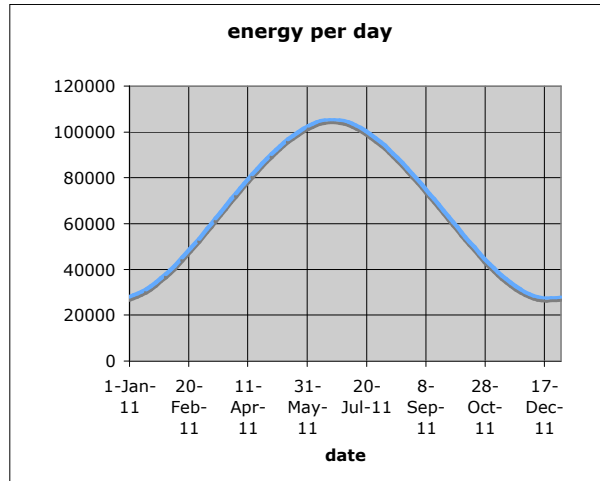
Obviously the student will know neither the sine function nor radians ... he has worked out some effective sizes by scale drawing and so should be content that if he had the time he could continue to do this.

The other required formula to put in is entered in F2 and filled down viz. $=C2*E2$
... and if total energy in the year is examined, then it is more effective if the total of column F is divided by 1000 000.

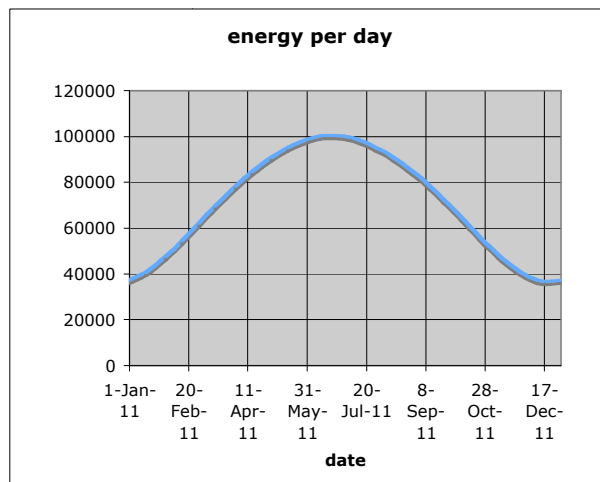
Pitch	Date	daylight minutes	altitude °	effective size	energy in day
53	1-Jan-11	426.2	11.2	90.03	38370.09
	15-Jan-11	457.3	13.1	91.43	41811.88
	1-Feb-11	517.8	17.1	94.03	48686.55
	15-Feb-11	577.6	21.5	96.36	55656.08
	1-Mar-11	641.2	26.6	98.36	63068.24
	15-Mar-11	706.3	32	99.62	70356.25
	1-Apr-11	785.4	38.7	99.96	78505.43
	15-Apr-11	849.8	43.9	99.28	84361.21
	1-May-11	920.5	49.2	97.74	89967.88
	15-May-11	976.6	53	96.13	93876.82
	1-Jun-11	1029.7	56.2	94.44	97244.01
	15-Jun-11	1052.7	57.4	93.73	98670.80
	1-Jul-11	1049.1	57.3	93.79	98389.24
	15-Jul-11	1020.6	55.7	94.72	96672.28
	1-Aug-11	963.8	52.2	96.5	93005.07
	15-Aug-11	906.6	48.2	98.1	88930.12
	1-Sep-11	831.5	42.4	99.56	82779.32
	15-Sep-11	767.8	37.2	100	76776.20
	1-Oct-11	694.3	31	99.45	69052.97
	15-Oct-11	630.4	25.7	98.06	61816.31
	1-Nov-11	555.0	19.8	95.53	53019.54
	15-Nov-11	498.3	15.7	93.17	46421.52
	1-Dec-11	446.7	12.4	90.92	40611.03
	15-Dec-11	421.8	11	89.88	37906.64
	31-Dec-11	424.6	11.1	89.96	38190.73

For different values of pitch we get different patterns of supply:

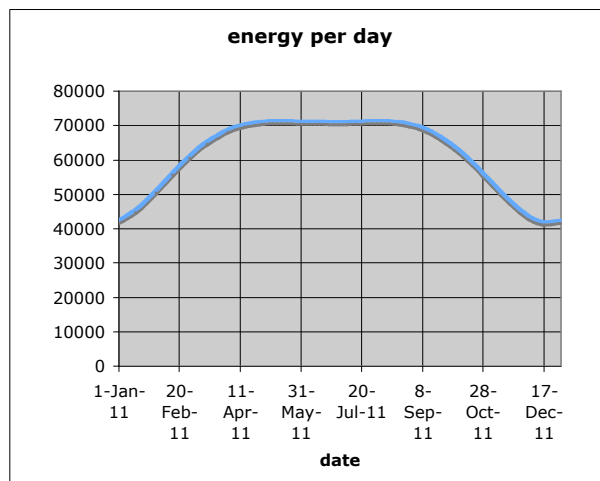
At 30°



at 50°



at 80°



Comments should include max and min attained and how sustained the supply is at 80° compared with 30° and similar ...

(ii) Find the value of the pitch which maximizes the annual output of energy.

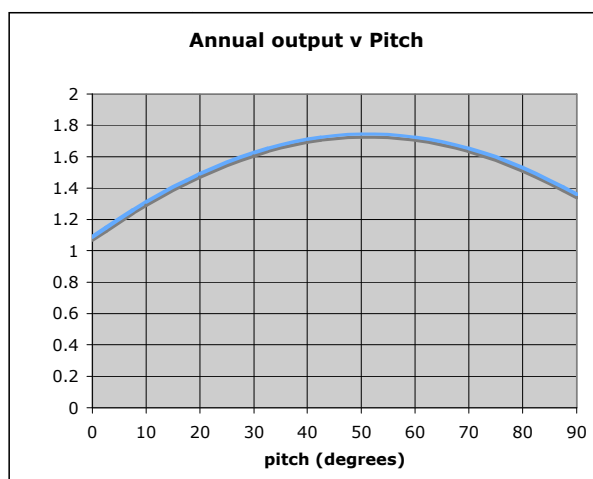
At each change of pitch the total annual energy is noted (in millions of units). This would be a good opportunity to use scientific notation ... 1.091×10^6 being better than 1091000 or 1.091 million.

This produces the table below. There is an obvious maximum around 50°.

Further refinement can be used to get an answer to any degree of accuracy desired.

pitch (°)	Annual energy ($\times 10^6$)
0	1.091
10	1.311
20	1.491
30	1.626
40	1.711
50	1.744
60	1.725
70	1.653
80	1.53
90	1.36

A graph highlights the maximum.



(iii) Write a short report, with charts and tables, which concludes with a recommendation for best pitch size in Glasgow.

After discussion, a report can be written taking full advantage of the table and chart making facilities of Excel and of the ease with which they can be cut and pasted into Word documents.

The report should include a recommended pitch-size. It should also include a caveat about what factors were not included in the consideration e.g. the altitude of the sun changes throughout the day.

Further Notes

- 1 I've recommended 4 separate periods for this. These may or may not be sequential. The learning potential in each Query is high. Judiciously spread out, the materials can be used effectively to introduce various aspects of the course including, geometry, scientific notation and an 'embryonic' trigonometry.
- 2 I really recommend the use of technology when making tables, graphs and writing reports. A lot can be learned without putting the pupil off through tedious repetitive graph drawing. It also produces a 'professional' finish of which the student will be proud.
- 3 An Excel spreadsheet is available with daily data pertinent to the project. This is set in Glasgow. It will differ for different locations throughout the country. The data has been cut and pasted, and edited from the web. The tab entitled 'Year' holds summary data which has been used to create the charts above. There are a lot of investigations still hidden within these tables. They also provide the basis of a great investigation into sine waves at a later date.
- 4 For the purposes of setting scales in graphs with a horizontal axis of dates we must be aware that the computer converts every date into a number. We don't need to know how it does it, but for making the curve start and finish at the 'right' places, we need the information that Jan 1st 2011 converts to 39082 and the 31st Dec 2011 converts to 39446.

If you wish the number corresponding to any date, say you wish 21st October 2012, you must type into some cell, =DATE(2012,10,21). This will return the required number 39741. (You may have to format the cell as a number, otherwise it sometimes returns the date!)