

## Flooding Prediction

If this topic is being covered in other subjects then the main function of mathematics might be in the modelling of a flood situation with a view to estimating the risk in a particular area.

There are many factors involved in flooding situations including rates of flow, permeability, catchment, drainage – so it may be logical to model it by analogy.

Learners prior knowledge of the learning context should be elicited to help their thinking.

When considering this scenario as an interdisciplinary activity, be careful to initiate each lesson by approaching the concept of flooding by analogy.

Modelling by analogy involves taking a topic of which the students are expected to have no direct experience and supporting them in developing a feel for the 'mechanisms' involved, by drawing parallels to a situation with which they are familiar.

Each analogy will bring with it different aspects of the problem of measuring the risk of flooding and the suggested remedy. In each scenario, there are various levels of approach, as indicated in the table below. To offer additional challenge, some more advanced algebra content is included at the end of the article.

| In the classroom | Suggested time | Brief outline                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | Links to key CfE E&Os                                   |
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| (1) the bath     | 2 hrs          | <ul style="list-style-type: none"> <li>• Introduction: compare the flooding problem with that of a bath filling when the taps are on and the plug is out.</li> <li>• Some problems with integer solutions (see Problems 1)</li> <li>• Use algebraic approaches to examine the problem in general ... thus enabling a change of context.</li> <li>• Compare these problems to that of a flooding area: discuss where the bath problem parallels the flooding situation; consider where the data might come from to analyse the flooding situation</li> <li>• Create problems using flooding scenarios that lead to the point where flooding might occur.</li> <li>• Give specifications of possible remedies such as overflow conduits and flood defences (dykes, levees)</li> </ul> <p>Introduction: compare the flooding problem with that of a crush zone in the school where the flow of people entering the zone is controlled by changing the number of doors which are open; the number of exits (corridors) is fixed ... cite some famous disasters where the problem was underestimated e.g. Glen Cinema Disaster or the Ibrox disaster.</p> | <p><b>Aspects of :</b><br/> <b>MNU 4-03a; 4-22a</b></p> |

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| (2) the atrium | 2 | <ul style="list-style-type: none"> <li>• begin an investigation with given data that explore the situation by varying the number of ingresses and keeping egresses fixed. (See Exercise 2)</li> <li>• Examine the point where a crush becomes a danger. Show that, because the population is finite, the danger situation only has to be contained for a limited time. This can be done by graphing 'contents' of crush zone or by using algebra.</li> <li>• In school, collect real data experimentally by timing how long it takes e.g. 30 pupils to enter a doorway or leave an atrium through a corridor. Thereafter, proportion can be considered.</li> <li>• Use algebra to generalise the problem: in association with spreadsheets, this will lead to examining the behaviour of the situation graphically.</li> <li>• Consider the relationship of this situation to that of the flooding problem, especially a consideration of a flood zone (c.f. crush zone)</li> <li>• Give specification of such a flood zone viz area and shape, consider how building can take place at the edge of the flood zone if the permeability is considered ... don't pave over surfaces that can accommodate flow.....</li> </ul> |  |
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| (3) the motorway | 2 | <ul style="list-style-type: none"> <li>• Introduction: compare the flooding problem to that encountered on a motorway viz. When travelling at 70 mph, you come across a traffic jam/tailback. After 10 minutes of 'crawling' you get to the front of the queue to find traffic speeding back up to 70mph never having encountered a reason for the jam.</li> <li>• The stopping distance is defined to be the safe distance between cars and is a function of the speed of the traffic. At 70 mph this should be 6 car-lengths, assuming a car is about 4 m long. Safe following distance is bigger than stopping distance. How long would it take to start getting a tailback? Provide support materials or, data sheets or internet access to encourage personal research. Note that at the time observations start in an area that some of the capacity of the area is already taken up.</li> <li>• Relate this scenario to the flood problem: can any of the motorway solutions e.g. reduced speed limit or extra lanes be applied to the flooding scenario?</li> <li>• Select a suitable local area that is prone to flooding. Get copies of ordnance survey maps showing catchment areas and contour lines.</li> <li>• Students can interpret map by (i) making drawings of cross-sections of 'valleys', by using contours.</li> <li>• Interpret the gradients to show direction of flow of surface water. Identify all the sources of water into the target area.</li> </ul> |  |
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| (4) Risk of Flooding | 2 | <ul style="list-style-type: none"> <li>• Calculate the impact on the area of one inch of rain falling in the catchment area. Consider e.g. volume of water is expected to flow in? ; rate at which it is flowing out?; rate at which the area is filling; will it reach capacity before all the expected estimated volume of water has passed through.</li> <li>• Then consider two inches of rain etc to discover the point where warnings should be given.</li> <li>• Rates of flow in specific areas might be found on the internet as can average rainfall. These can be used to construct a chart with calculated likelihood of flooding throughout the year.</li> <li>• Investigate the factors which might affect the premiums of an insurance policy against flooding.</li> </ul> |  |
| 5                    | 1 | <ul style="list-style-type: none"> <li>• In groups, learners can evaluate the different ways of flood control w.r.t. cost of defence against cost of damage</li> <li>• Produce a summary report.</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |  |

## Graded problems 1

1 How might you estimate the capacity of a bath?

[Hint: estimate as a cuboid:  $L \times B \times D$  ... all internal measurements]

Research: Find the capacity of a standard bath in litres and the usual amount of water used when a bath is run.

- 2
  - (a) If a bath of capacity 215 litres can be filled by a hot tap in 35 minutes at what rate is the water being supplied by the tap?
  - (b) The bath can be filled by a cold tap in 14 minutes. If both taps are on, at what rate is the bath filling and how long will it take the bath to fill?
  - (c) When the plug is pulled out, this full bath will empty in 15 minutes.  
At what rate does the drain get rid of the water?
- 3 Repeat question 2 when a bath of capacity 225 litres can be filled from the hot tap in 18 minutes, by the cold tap in 9 minutes and can be emptied in 12 minutes, with the taps off and the plug out.
- 4 What conditions must be met for the bath to fill if both taps are on and the plug is out?
- 5 A man starts to run a bath; he turns both taps on and then is called away to the phone before he has put in the plug.
  - (a) At what rate will the bath fill?
  - (b) How long can he afford to be on the phone before the bath overflows?
  - (c) What design feature of a bath tries to accommodate such an eventuality?
  - (d) What rate of flow must it accommodate?
  - (e) What volume of water will pass through the bath by the time it reaches the overflow point?

## Exercise 2

A school has 1200 pupils, all in the playground.

When the bell rings the pupils come in by one of six doors which each lead into an atrium.

Pupils leave this atrium through corridors which take the pupils to their classes.

Ideally we want all the pupils to get to class within 5 minutes of the bell ringing without creating the dangerous situation of a crush.

A crush will occur if the pupils outside continue to come in despite the fact that atrium is filled to capacity.

### Building a model.

Suppose each door will admit 30 pupils per minute into the atrium and that the corridor system will take 40 out per minute.

Suppose two doors are open.

- (a) How long will it take to get all the pupils into school?
- (b) How many pupils will be in the atrium by this time?
- (c) Comment on
  - (i) the practicalities of your answer to (a)
  - (ii) the minimum capacity of the atrium for the sake of safety.

### Answer

- (a) With two doors open 60 pupils enter per minute. It will take  $1200 \div 60 = 20$  minutes to get everyone into the building.
- (b) The rate that the atrium is filling  $60 - 40 = 20$  pupils per minute.  
In 20 minutes there would be  $20 \times 20 = 400$  pupils in atrium.
- (c)
  - (i) It's not acceptable to lose 20 minutes of a period.
  - (ii) To avoid a crush, the atrium has to be designed to hold 400 pupils.

Using the above worked model , explore the situation when 3, 4, 5 or 6 doors are open.

How might the real data be found to consider the design problem?

Describe some experiments that might be practical to explore the situation.

### Exercise 3

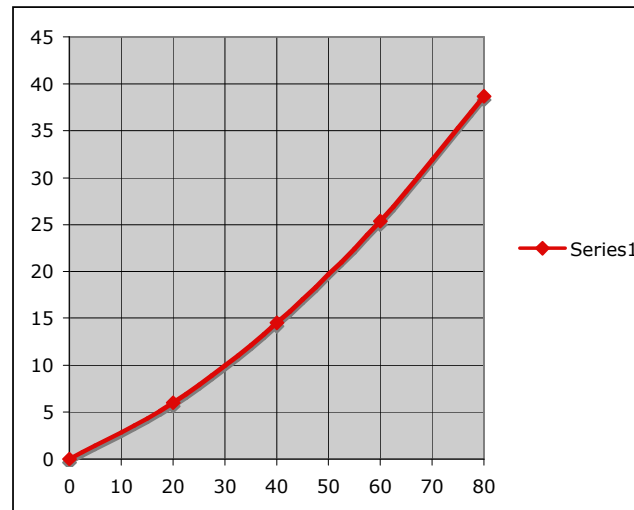
Building a model:

Examine the highwaycode and find a relationship between speed and stopping distance in car-lengths.

(a) get figures

(b) use a spreadsheet to draw a graph to permit interpolation

| speed (mph) | safe following distance (car-lengths) |
|-------------|---------------------------------------|
| 0           | 0                                     |
| 20          | 6                                     |
| 40          | 14.5                                  |
| 60          | 25.4                                  |
| 80          | 38.7                                  |



## Examine the following argument

A car-length is estimated as 12 feet ... 440 car lengths to a mile ... the measured mile's capacity.  
In a particular mile, with cars travelling safely at 70 mph, each car occupies its own length plus its safe following distance ...  $1 + 33 = 34$  car-lengths,  
so there should be  $440 \div 34 = 13$  cars in a mile of motorway lane ... initial content ... and  $440 - 13 \text{ cars} = 427$  cars available capacity.  
Cars are entering this mile at 70 mph =  $(70 \times 13)$  cars per 60 minutes = 13.7 cars per minute.

If, at the end of the mile a car slows to 1 mph, cars will start to leave the mile occupying a space of  $1 + 1 = 2$  car lengths (from graph).  
Each mile will contain  $440 \div 2 = 220$  cars.  
Cars leave the mile at 1 mph =  $(1 \times 220)$  cars per 60 minutes = 3.7 cars per minute.

So, the measured mile will fill at the rate of 10 cars per minute.  
The mile will fill in  $427/10 = 42.7$  minutes ... i.e the tailback will grow at 38 metres /minute.

Suppose the traffic is travelling at (i) stopping distance (ii) braking distance. Explore both scenarios.

How does such a scenario relate to the problem of flooding.  
How might the solutions to one problem relate to the solutions of the other?

## Appendix 3

Some algebraic consideration can be given to each scenario examined and the results used to develop useful spreadsheets. Done properly Excel spreadsheets could be used to simulate situations or investigate 'what if' situations.

Purely as an example ... consider a bath, hot tap fills in  $x$  minutes, cold tap in  $y$  minutes and empties in  $z$  minutes:

$$\text{rate of flow of hot tap} = \frac{V}{x} \text{ litres/min}$$

$$\text{rate of flow of cold tap} = \frac{V}{y} \text{ litres/min}$$

$$\text{rate of flow through plug} = \frac{V}{z} \text{ litres/min}$$

$$\text{Resultant fill rate} = V \left( \frac{1}{x} + \frac{1}{y} - \frac{1}{z} \right) \text{ litres/min ... any safety overflow that is installed must cope with this rate.}$$

$$\text{Time to fill bath, } T = \frac{V}{V \left( \frac{1}{x} + \frac{1}{y} - \frac{1}{z} \right)} = \frac{1}{\frac{1}{x} + \frac{1}{y} - \frac{1}{z}}$$

This relation has a neat symmetry when expressed as  $\frac{1}{T} = \frac{1}{x} + \frac{1}{y} - \frac{1}{z}$

**We can study such functions by exploring under what conditions can we produce integer solutions?**

If the bath will fill from all sources in  $a$  minutes with the plug in then we want  $\frac{V}{a} = \frac{V}{x} + \frac{V}{y}$

Let  $x = a + a_1$  and  $y = a + a_2$ . (We know  $x > a$  and  $y > a$ )

$$\frac{1}{a} = \frac{1}{x} + \frac{1}{y}$$

$$\Rightarrow \frac{1}{a} = \frac{1}{a + a_1} + \frac{1}{a + a_2}$$

$$\Rightarrow (a + a_1)(a + a_2) = a(a + a_2) + a(a + a_1)$$

$$\Rightarrow a^2 + a(a_1 + a_2) + a_1a_2 = 2a^2 + a(a_1 + a_2)$$

$$\Rightarrow a^2 = a_1a_2$$

So if we want to make up a question where two integer flows,  $x$  and  $y$ , have to combine to produce an integer resultant,  $a$ , then

- make up a value for  $a$  ...say, 6
- square and factorise into complementary pairs:  $6^2 = 36$ : (1, 36), (2, 18), (3, 12), (4, 9), (6, 6).
- add  $a = 6$  to each number: (7, 42), (8, 26), (9, 18), (10, 15), (12, 12) ... to give you , in this case, five sets of values suitable for easy questions.

e.g. If the hot tap fills in 10 mins and the cold in 15 mins then both together will fill it in 6 mins since  $\frac{1}{6} = \frac{1}{10} + \frac{1}{15}$

I'm sure the science department could devise circuits that mimic these flows using resistors in parallel.

## Appendix 5

### Some additional notes on analogy:

#### (1) Bath analogy:

This is the old chestnut which is often ridiculed as an impractical situation but which in reality comes close to illustrating the mechanisms involved in flooding.

A bath of volume  $V$  litres will fill in  $x$  minutes if only the hot tap is running and the plug is in, will fill in  $y$  minutes if only the cold tap is running and the plug is in and will empty in  $z$  minutes if the plug is out and the taps are off.

- (i) What are the conditions needed so that the bath starts to fill with both taps on and the plug out?
- (ii) Under these conditions how long will it be,  $T$  minutes, before the bath overflows?
- (iii) If the taps are fed from a header tank of fixed volume,  $W$  litres, will this drain before the bath overflows?
- (iv) What measures can be taken to avoid overflow onto the floor?
  - (a) create an overflow pipe ... what rate of flow must it accommodate?
  - (b) build up the sides of the bath ... what capacity must be accommodated?

This can be physically tried out in the science lab where rates of flow can be calculated and controlled; where the permeability can be examined by filling the 'bath' with sand etc.

The analogy would then have to be studied to see how the model relates to flooding.

|               |   |                                     |
|---------------|---|-------------------------------------|
| bath          | ↔ | area under consideration            |
| taps          | ↔ | catchment                           |
| plug          | ↔ | drainage                            |
| header        | ↔ | total volume passing through system |
| overflow pipe | ↔ | artificial drainage                 |
| build-up      | ↔ | reinforced embankment               |

## (2) Crush zones

Architects have to take flow into consideration when designing buildings or stadia where there is a potential for overcrowding and crush.

When the school bell rings students enter the building through doors which allow all the pupils through in  $x$  minutes. They enter into what's called a crush zone. The students disperse along the corridors to their classrooms and this would happen in  $y$  minutes.

For what values of  $x$  and  $y$  does a dangerous situation occur?

What remedies can be employed?

The system only has to cope for the time it takes the whole school to get to class ... so how big should the crush zone be given that the students are going into it faster than they are going out?

Here we have to consider that a discrete variable is being used to model a continuous variable.  
This will allow counting to be used as a measure of the variable though some aspects will be lost.

Here the point at which crushing occurs is analogous to the point at which flooding occurs. The solutions to this problem hint at other solutions to the flooding problem. One might close an entrance thereby controlling the inflow (c.f. Thames Barrier). One might make emergency break-out zones in the crush zone (c.f. flood plain).

For this situation, data might be collected by counting the rate at which people enter the zone and at which they disperse. Charts can be drawn up showing *people in zone* over *time* to help identify the point at which *crush* occurs.

The analogy would then have to be studied to see how the model relates to flooding.

|                      |   |                                                                                                                |
|----------------------|---|----------------------------------------------------------------------------------------------------------------|
| zone                 | ↔ | area under consideration                                                                                       |
| entrance             | ↔ | catchment                                                                                                      |
| corridors            | ↔ | drainage                                                                                                       |
| population of school | ↔ | total volume passing through system                                                                            |
| break-out            | ↔ | flood plain                                                                                                    |
| inflow control       | ↔ | barrier (note the barrier only works because there is a limit to the expected time that flow must be endured). |

### (3) Traffic Jams

Traffic jams occur if (i) there is a bottleneck or (ii) flow out of a section of the motorway is less than flow in. By S2 students will have encountered being caught in a traffic jam and finding, when they get to the end, there is no apparent reason for the hold up.

Consider a busy 1-mile stretch of motorway with cars driving safely (two car-lengths apart). Cars enter this stretch at 70 mph. At the other end, cars have to drop to 10 mph for 10 seconds then it clears up. What effect will this have on the 1-mile stretch assuming cars?

Again discrete variables are being used the model continuous variables. The flood area/bath is now a 1-mile stretch of motorway. When it is being considered, the system is already in operation with the stretch containing cars ... at  $\frac{1}{3}$  of its capacity.

Students will have to consider what information they require (e.g. car-length) to work out flow rates. None of the remedies associated with either bath or crowd control are appropriate. The only 'remedy' here is to let the traffic back up in the knowledge that it will clear.

Again the analogy would then have to be studied to see how the model relates to flooding.

|                        |   |                          |
|------------------------|---|--------------------------|
| stretch of road        | ↔ | area under consideration |
| stretch before         | ↔ | catchment                |
| stretch after          | ↔ | drainage                 |
| volume passing through | ↔ | effectively no limit     |

### Getting Facts from the Actual Situation

A suitable area in the neighbourhood should be selected as the target area.

By looking at contour maps the catchment area can be identified.

If an inch of rain falls on this catchment area (including the area under consideration), how much water is that?

How might we measure the amount of water flowing out of the area?

Can we look up historical records to find the rate of rainfall that has resulted in flooding in the past?

Can we use that to create a predictive model for the area ... relate inches of rainfall to probability of flooding?

If there is flooding, what area of the map will be affected?

The interpretation of maps and plans, of graphs and charts, and of rates are important. The awareness of local variables such as gradients, permeability of soil, water tables and rock types and that they are not covered by the model is also important.